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Within the scope of the relativistic quantum theory for electron-laser interaction in a medium and using the resonant approximation for the two degenerated states of an electron in a monochromatic radiation field [1] a nonperturbative solution of the Dirac equation (nonlinear over field solution of the Hill type equation) are obtained. The multiphoton cross sections of electrons coherent scattering on the plane monochromatic wave at the Cherenkov resonance are obtained taking into account the specificity of induced Cherenkov process [1, 2] and spin-laser interaction as well. In the result of this resonant scattering the electron beam quantum modulation at high frequencies occurs that corresponds to a quantity of an electron energy exchange at the coherent reflection from the "phase lattice" of slowed plane wave in a medium. So, we can expect to have a coherent X-ray source in induced Cherenkov process, since such beam is a potential source of coherent radiation itself.

I. INTRODUCTION

As is known the coherent interaction of electrons with a plane monochromatic wave in a dielectric medium can be described as a resonant scattering of a particle on the "phase lattice" of a traveling wave similar to the Bragg scattering of the particle on the crystal lattice [1], [2]. The latter is obvious in the frame of reference (FR) of rest of the wave. Since the index of refraction of a medium $n > 1$ ($n(\omega) \equiv n$ as the wave is monochromatic) in this FR there is only a static periodic magnetic field and an elastic scattering of a particle takes place. The law of conservation for Cherenkov process taking into account the quantum recoil translates into the Bragg resonance condition between the de Broglie wave of the particle and this static periodic structure. Hence, in induced Cherenkov process the interaction resonantly connects two states of the particle which are degenerated over the longitudinal momentum with respect to the direction of the wave propagation: the states with longitudinal momenta p_x - of the incident particle and the states with longitudinal momenta $p_x + \ell \hbar k$ - of scattered "Bragg" particle, as far as the conservation law of this process is $|p_x| = |p_x + \ell \hbar k|$ (ℓ - number absorbed or radiated photons with a wave vector $k = k_x$). The latter is the same as the Bragg condition of coherent elastic scattering. Therefore, in stimulated Cherenkov process no matter how weak the wave field is the usual perturbation theory is not applicable because of such degeneration of the states. So, the interaction near the resonance is necessary to describe by the secular equation [1]. The latter, in particular, reveals zone structure of the particle states in the field of transverse electromagnetic (EM) wave in a dielectric medium [1], [2]. Note that the application of the perturbation theory ignoring the mentioned degeneration in this process has reduced to essentially incorrect results which have been elucidated in the paper [3].

In the present work the case of strong radiation field is considered within the scope of the relativistic quantum theory for electron-laser interaction in a medium. Using the resonance approximation for the above mentioned two degenerated states in a monochromatic radiation field [1] a nonperturbative solution of the Dirac equation (nonlinear over field solution of the Hill type equation) are obtained. The multiphoton probabilities of free electrons coherent scattering on a strong monochromatic wave at the Cherenkov resonance are counted, taking into account the above mentioned specificity of induced Cherenkov process [1], [2] and spin-laser interaction as well. In the result of this resonant scattering the electron beam quantum modulation at high frequencies occurs that corresponds to the electron energy exchange at the coherent reflection from the "phase lattice" of slowed wave in a medium. So, we can expect to have, in principle, a coherent X-ray source in induced Cherenkov process, since such-quantum modulated beam is a potential source of coherent radiation itself.

II. NONLINEAR SOLUTION OF THE DIRAC EQUATION FOR ELECTRON IN STRONG EM RADIATION FIELD IN A MEDIUM

In this section we shall solve Dirac equation for a spinor particle in the given radiation field in a medium

$$i\frac{\partial\Psi}{\partial t} = \left[\widehat{\alpha}(\widehat{\mathbf{p}} - e\mathbf{A}(\tau)) + \widehat{\beta}m\right]\Psi, \quad (1)$$

where

$$\widehat{\alpha} = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}; \quad \widehat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2)$$

are the Dirac matrices, with the σ Pauli matrices, m and e are the mass and charge of a particle respectively (here we set $\hbar = c = 1$), $\widehat{\mathbf{p}} = -i\hbar\nabla$ - the operator of the generalized momentum $\mathbf{A} = \mathbf{A}(t - nx/c)$ -is the vector potential of a linearly polarized plane wave propagating in the OX direction in a medium:

$$\mathbf{A} = \{0, A_0(\tau) \cos \omega\tau, 0\}; \quad \tau = t - nx/c. \quad (3)$$

We shall assume that EM wave is adiabatically switched on at $\tau = -\infty$ and switched off at $\tau = +\infty$ ($\mathbf{A}(\tau = \mp\infty) = 0$).

To solve the problem it is more convenient to pass to the FR of rest of the wave (R frame moving with the velocity $V = 1/n$). As it is noticed, in this FR there is only static magnetic field which will be described according to (3) by the following vector potential

$$\mathbf{A}_R = \{0, A_0(x') \cos k'x', 0\}, \quad (4)$$

where

$$k' = \omega\sqrt{n^2 - 1}. \quad (5)$$

The wave function of a particle in R frame is connected with the wave function in laboratory frame Λ by the Lorentz transformation of the bispinors

$$\Psi = \widehat{S}(\vartheta)\Psi_R, \quad (6)$$

where

$$\widehat{S}(\vartheta) = ch\frac{\vartheta}{2} + \alpha_x sh\frac{\vartheta}{2}; \quad th\vartheta = V = \frac{1}{n} \quad (7)$$

is the transformation operator. For Ψ_R we have the following equation

$$i\frac{\partial\Psi_R}{\partial t'} = \left[\widehat{\alpha}(\widehat{\mathbf{p}}' - e\mathbf{A}_R(\mathbf{x}')) + \widehat{\beta}m\right]\Psi_R. \quad (8)$$

Since the interaction Hamiltonian does not depend on the time and transverse (to the direction of the wave propagation) coordinates the eigenvalues of the operators \widehat{H}' , \widehat{p}'_y , \widehat{p}'_z are conserved: $E' = \text{const}$, $p'_y = \text{const}$, $p'_z = \text{const}$ and the solution of Eq.(8) can be represented in the form of a linear combination of free solutions of the Dirac equation with amplitudes $a_i(x')$ depending only on x' :

$$\Psi_R(\mathbf{r}', t') = \sum_{i=1}^4 a_i(x') \Psi_i^{(0)}. \quad (9)$$

Here

$$\begin{aligned} \Psi_{1,2}^{(0)} &= \left(\frac{E' + m}{2E'}\right)^{\frac{1}{2}} \left[\begin{array}{c} \varphi_{1,2} \\ \frac{\sigma_x p'_x + \sigma_y p'_y}{E' + m} \varphi_{1,2} \end{array} \right] \exp[i(p'_x x' + p'_y y' - E' t)], \\ \Psi_{3,4}^{(0)} &= \left(\frac{E' + m}{2E'}\right)^{\frac{1}{2}} \left[\begin{array}{c} \varphi_{1,2} \\ \frac{-\sigma_x p'_x + \sigma_y p'_y}{E' + m} \varphi_{1,2} \end{array} \right] \exp[i(-p'_x x' + p'_y y' - E' t)], \end{aligned} \quad (10)$$

where

$$p'_x = (E'^2 - p_y'^2 - m^2)^{\frac{1}{2}}, \quad \varphi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (11)$$

The solution of Eq. (8) in the form (9) corresponds to an expansion of the wave function in a complete set of the wave functions of a electron with certain energy and transverse momentum p'_y (with longitudinal momenta $\pm(E'^2 - p_y'^2 - m^2)^{\frac{1}{2}}$ and spin projections $S_x = \pm\frac{1}{2}$). The latter are normalized to one particle per unit volume. Since there is symmetry with respect to the direction \mathbf{A}_R (the OY axis) we have taken, without loss of generality, the vector \mathbf{p}' in the XY plane ($p'_z = 0$).

Substituting Eq.(9) into Eq.(8) then multiplying by the Hermitian conjugate functions and taking into account (10) and (2) we obtain a set of differential equations for the unknown functions $a_i(x')$. The equations for a_1 , a_3 and a_2 , a_4 are separated and for these amplitudes we have the following set of equations

$$\begin{aligned} p'_x \frac{da_1(x')}{dx'} &= iep_y A_y(x') a_1(x') - eA_y(x') (p'_x - ip'_y) \cdot a_3(x') \exp(-2ip'_x x'), \\ p'_x \frac{da_3(x')}{dx'} &= -iep_y A_y(x') a_3(x') - eA_y(x') (p'_x + ip'_y) \cdot a_1(x') \exp(2ip'_x x'). \end{aligned} \quad (12)$$

A similar set of equations is also obtained for the amplitudes $a_2(x')$ and $a_4(x')$. For simplicity we shall assume that before the interaction there are only electrons with specified longitudinal momentum and spin state, i. e.

$$|a_1(-\infty)|^2 = 1, \quad |a_3(+\infty)|^2 = 0, \quad |a_2(-\infty)|^2 = 0, \quad |a_4(+\infty)|^2 = 0. \quad (13)$$

From the condition of conservation of the norm we have

$$|a_1(x')|^2 - |a_3(x')|^2 = const \quad (14)$$

and the probability of reflection is $|a_{3,4}(-\infty)|^2$.

The application of the following unitarian transformation

$$\begin{aligned} a_1(x') &= b_1(x') \exp \left(i \frac{ep'_y}{p'_x} \int_{-\infty}^{x'} A_y(\eta) d\eta - i \frac{\vartheta'}{2} \right) \\ a_3(x') &= b_3(x') \exp \left(-i \frac{ep'_y}{p'_x} \int_{-\infty}^{x'} A_y(\eta) d\eta + i \frac{\vartheta'}{2} \right), \end{aligned} \quad (15)$$

simplifies Eq.(12). Here ϑ' is the angle between the momentum of electron and the direction of the wave propagation in the R frame. The new amplitudes $b_1(x')$ and $b_3(x')$ satisfy the same initial conditions: $|b_1(-\infty)|^2 = 1$, $|b_3(+\infty)|^2 = 0$, according to (13).

From Eq.(12) and Eq.(15) for the $b_1(x')$ and $b_3(x')$ we obtain the following set of equations

$$\begin{aligned} \frac{db_1(x')}{dx'} &= -f(x') b_3(x') \\ \frac{db_3(x')}{dx'} &= -f^*(x') b_1(x') \end{aligned} \quad (16)$$

where

$$f(x') = \frac{eA_y(x')p'}{p'_x} \exp \left(-2ip'_x x' - i \frac{2ep_y}{p'_x} \int_{-\infty}^{x'} A_y(\eta) d\eta \right); \quad p' = \sqrt{p_y'^2 + p_x'^2} \quad (17)$$

Using the following expansion by the Bessel functions

$$\exp(-i\alpha \sin kx) = \sum_{N=-\infty}^{\infty} J_N(\alpha) \exp(-iNkx),$$

we can reduce Eq. (16) to the following form

$$\frac{db_1(x')}{dx'} = - \sum_{N=-\infty}^{\infty} f_N \exp[-i(2p'_x - Nk)x'] b_3(x')$$

$$\frac{db_3(x')}{dx'} = - \sum_{N=-\infty}^{\infty} f_N \exp[i(2p'_x - Nk)x'] b_1(x') \quad (18)$$

where

$$f_N = \frac{p'}{2p'_y} N k' J_N \left(2\xi \frac{m}{p'_x} \frac{p'_y}{k} \right); \quad \xi = eA/m \quad (19)$$

III. RESONANT APPROXIMATION FOR TRANSITION AMPLITUDES

Because of conservation of particle energy and transverse momentum (in R frame) the real transitions in the field will occur from a p'_x state to the $-p'_x$ one and, consequently, the probabilities of multiphoton scattering will have a maximal values for the resonant transitions

$$2p'_x = sk', \quad (s = \pm 1, \pm 2 \dots) \quad (20)$$

The latter expresses the condition of exact resonance between the electron de Broglie wave and the incident "wave lattice". In the Λ frame inelastic scattering takes place and the Eq.(20) corresponds to the well known Cherenkov conservation law

$$\frac{2E(1 - nv \cos \vartheta)}{(n^2 - 1)} = s\omega \quad (21)$$

where ϑ is the angle between the electron momentum and the wave propagation direction in the Λ frame (the Cherenkov angle), v and E are the electron velocity and energy.

So, we can utilize the resonant approximation keeping only resonant terms in the Eq.(18). Generally, in this approximation, at detuning of resonance $|\delta_s| = |2p'_x - sk'| \ll k'$, we have the following set of equations for the certain s -photon transition amplitudes $b_1^{(s)}(x')$ and $b_3^{(s)}(x')$:

$$\frac{db_1^{(s)}(x')}{dx'} = -f_s \exp[-i\delta_s x'] b_3^{(s)}(x')$$

$$\frac{db_3^{(s)}(x')}{dx'} = -f_s \exp[i\delta_s x'] b_1^{(s)}(x') \quad (22)$$

This resonant approximation is valid for the slow varying functions $b_1^{(s)}(x')$ and $b_3^{(s)}(x')$, i. e. at the following condition

$$\left| \frac{db_{1,3}^{(s)}(x')}{dx'} \right| \ll \left| b_{1,3}^{(s)}(x') \right| \cdot k'. \quad (23)$$

At first we shall solve the case of exact resonance ($\delta_s = 0$). According to the boundary conditions (14) we have the following solutions for the amplitudes

$$b_1^{(s)}(x') = \frac{ch \left[\int_{x'}^{\infty} f_s d\eta \right]}{ch \left[\int_{-\infty}^{\infty} f_s d\eta \right]}, \quad b_3^{(s)}(x') = \frac{sh \left[\int_{x'}^{\infty} f_s d\eta \right]}{ch \left[\int_{-\infty}^{\infty} f_s d\eta \right]} \quad (24)$$

and for the reflection coefficient

$$R^{(s)} = \left| b_3^{(s)}(-\infty) \right|^2 = th^2 [f_s \Delta x'] \quad (25)$$

where $\Delta x'$ is the coherent interaction length. The reflection coefficient in the laboratory frame of reference is the probability of absorption at $v < 1/n$ or emission at $v > 1/n$. The latter can be obtained expressing the quantities f_s and $\Delta x'$ by the quantities in this frame since the reflection coefficient is Lorentz invariant. So

$$R^{(s)} = th^2 [F_s \Delta \tau] \quad (26)$$

where

$$F_s = \left[\frac{(1 - nv \cos \vartheta)^2}{n^2 - 1} + v^2 \sin^2 \vartheta \right]^{1/2} \frac{s\omega}{2v \sin \vartheta} J_s \left(\frac{\xi}{n^2 - 1} \cdot \frac{2mv \sin \vartheta}{\omega(1 - nv \cos \vartheta)} \right) \quad (27)$$

and $\Delta \tau$ for actual cases is the laser pulse duration in the Λ frame. The condition of applicability of this-resonant approximation (23) is equivalent to the condition

$$|F_s| \ll \omega, \quad (28)$$

which restricts as the intensity of the wave as well as the Cherenkov angle. Besides, to satisfy the condition (28) we must take into account the very sensitivity of the parameter F_s towards the argument of Bessel function, according to Eq.(27). For the wave intensities when $F_s \Delta \tau \gtrsim 1$ the reflection coefficient is in the order of unit which can occur for the large number of photons $s \gg 1$ when the argument of Bessel function $Z \sim s \gg 1$ in Eq.(27) (according to asymptotic behavior of Bessel function $J_s(Z)$ at $Z \simeq s \gg 1$).

Let us estimate the reflection coefficient of an electron from the laser pulse or the most probable number of absorbed/emitted photons due to resonance interaction in induced Cherenkov process. For the typical values of experimental parameters of this process in the gaseous medium with the index of refraction $n - 1 \sim 10^{-4}$, at the initial electron energy $E \sim 50 \text{ Mev}$ and Cherenkov angle $\vartheta \sim 1 \text{ mrad}$, during the "Bragg reflection" from Neodymium laser pulse ($\omega \Delta \tau \sim 10^2$, $\hbar\omega = 1.17 \text{ eV}$) with an intensity 10^{10} W/cm^2 ($\xi \sim 10^{-4}$) electron absorbs or emits about 10^5 photons.

For the off resonant solution, when $\delta_s \neq 0$, but $f_s^2 > \delta_s^2/4$ from Eq.(22) for $R^{(s)}$ the following expression we obtain

$$R^{(s)} = \frac{f_s^2}{\Omega_s^2} \frac{sh^2[\Omega_s \Delta x']}{1 + \frac{f_s^2}{\Omega_s^2} sh^2[\Omega_s \Delta x']} \quad (29)$$

where $\Omega_s = \sqrt{f_s^2 - \delta_s^2/4}$, which has the same behavior as in the case of exact resonance. In opposite case when $f_s^2 \leq \delta_s^2/4$ the reflection coefficient is a oscillating function on interaction length.

During the coherent interaction with EM wave the quantum modulation of particles beam density occurs too which in difference to classic one after the interaction remains unlimitedly long (for the monochromatic beam). This is a result of coherent superposition of particle states with various energy and momentum due to absorbed and emitted photons in the radiation field which is conserved after the interaction. The quantum modulated state of the particle leads to modulation of the beam density after the interaction at the frequency of the stimulating wave and its harmonics [4]. The density modulated particles beam can be used to generate spontaneous superradiation [5] The various radiation mechanisms of quantum modulated beams are investigated in the works [6]- [8].

In stimulated Cherenkov process the beam quantum modulation occurs if the particles wave packet size (Δx) is enough large: $\Delta x \gg \lambda$ (λ - is the radiation wavelength). In the opposite case the classic modulation or bunching of the beam takes place (klystron interaction scheme). From Eq. (9) and Eq. (26) for the electron wave function after the reflection from the wave pulse we have the superposition of incident and reflected electron waves (in the R frame)

$$\Psi_R = a_1(-\infty)\Psi_1^{(0)} + a_3(-\infty)\Psi_3^{(0)} \quad (30)$$

and in the result the probability density $\rho_R = \Psi_R^\dagger \Psi_R$ is modulated at the X-ray frequencies

$$\rho_R^{(s)} = 1 + th^2 [f_s \Delta x'] + 2 \left[1 - \frac{p_x'^2}{E'^2} \right] th [f_s \Delta x'] \cos(sk'x' - \varphi_0) \quad (31)$$

where

$$\left[1 - \frac{p_x'^2}{E'^2} \right] \sin \varphi_0 = \sin \vartheta'.$$

In the laboratory frame of the reference from Eq.(7) and Eq.(30) we have

$$\rho^{(s)} \simeq \frac{1}{\sqrt{n^2 - 1}} (1 + th^2 [F_s \triangle \tau] + 2th [F_s \triangle \tau] \cos(s\omega\tau - \vartheta')) \quad (32)$$

where it is taken into account that in actual case $|s\omega| \ll E$. As is seen from Eq.(30) the modulation depth is in the order of unit for the intensities when $F_s \triangle \tau \sim 1$ which can be satisfied for the moderate intensities of the laser radiation in the order of $10^{10} W/cm^2$.

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